# A Novel Correlative Interferometer Technique with Multi-Sample Diversity for Finding Direction of Satellites

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Abstract—A precise direction-finding technique is required even in low signal-to-noise ratio (SNR) regimes for wireless positioning applications using commercial satellites. In this paper, we propose a novel correlative interferometer (CI) technique that accurately estimates the direction-of-arrival (DoA) of multiple satellite signals incident on an array antenna by exploiting multiple time samples. The basic idea of the proposed technique is to jointly exploit multiple time samples of the received satellite signal in estimating DoAs under the presence of additive noise. The proposed CI technique can be regarded as a generalized version of the conventional CI technique since it reduces to the conventional CI technique when it applies to the case of a single time sample. Through computer simulations, we show that the proposed CI technique significantly outperforms the conventional CI and the multiple signal classification (MUSIC) techniques in terms of DoA estimation even in the low SNR regime.

*Index Terms*—6G, wireless positioning, direction-ofarrival (DoA), direction finding, array antenna, correlative interferometer (CI), multi-sample diversity.

#### I. INTRODUCTION

In next-generation mobile communication systems, wireless positioning and sensing are expected to become emerging technologies as new service applications [1]. To this end, direction-finding technologies for multiple signal sources have been actively developed, and direction-of-arrival (DoA) estimation techniques using array antennas have been receiving attention to achieve a high resolution and robust performance [2]. Furthermore, in recent, the potential for position, navigation, and timing (PNT) systems using commercial satellites such as low earth orbit (LEO) has also been suggested [3]. Hence, the DoA estimation using array antennas can be fully utilized for the LEO-PNT systems. In fact, DOA estimation has been already utilized for the global navigation satellite system (GNSS) receiver to classify spoofing signal which is a counterfeit signal for deception [4], [5].

Over the past decades, various array antenna-based DoA estimation algorithms have been studied. When an array antenna-based receiver estimates the DoA of multiple signal sources, the maximum likelihood estimator (MLE) is known as the optimal DoA estimation algorithm [6]. However, MLE method is somewhat impractical and requires very high computational complexity. On the other hand, multiple signal classification (MUSIC) is one of the well-known practical DoA estimation techniques using an array antenna by exploiting the orthogonality between the subspace of signal components and noise components in the covariance matrix of the received signal [7]. In addition, compressed sensing (CS)-based DoA

estimation algorithms using sparsity between incident directions are also being actively investigated [5], [8]. Moreover, there are studies that apply learning-based DoA estimation [9], [10], but the activation time is quite long because the model must be configured and learned in advance.

Correlative interferometer (CI) algorithm is a greedy DoA estimation technique that selects the most probable phase candidate using the difference between the phase of the received signal and all candidate phases and extracts the directional information of the selected phase [11]. That is, in wireless positioning using far-off satellite signals, the CI algorithm exploiting only the phase rather than the magnitude of the received signal might show a more accurate DoA estimation performance. However, most existing studies related to CI mainly require single-source environment or high SNR assumptions. If the receiver collects several samples and jointly exploits the spatial features of the received signal within each sample, improved DoA estimation performance with higher resolution can be achieved. Nonetheless, the CI algorithm has rarely been investigated on how it works when receiving multiple time-domain samples. Therefore, in this paper, we propose a novel CI technique with multi-sample diversity that extends the existing CI algorithm for the direction-finding of multiple signals using multiple time-domain samples. The basic idea is that by gathering samples over a number of times in the presence of additive noise, the receiver jointly explores the spatial characteristics of the received signal in each timedomain sample. Through computer simulations, we verify that the proposed CI technique elaborately estimates the DoA of multiple satellite signals in the low SNR regime.

## II. SYSTEM MODEL

In this paper, we consider an environment in which a single receiver equipped with a uniform linear array (ULA) with M antenna elements receives signals from K satellites as shown in Fig. 1. In addition, it is assumed that all satellites exist in line-of-sight (LoS) to the receiver and multipath signals are negligibly small or absent as in many related studies. Then, the received signal,  $\mathbf{y} \in \mathbb{C}^M$ ), at the receiver is given by

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w},\tag{1}$$

where  $\mathbf{A} (\in \mathbb{C}^{M \times K})$  denotes a matrix concatenating the steering vectors of each incident signal source, i.e.,  $\mathbf{A} = [\mathbf{a}(\theta_1)\cdots\mathbf{a}(\theta_k)\cdots\mathbf{a}(\theta_K)]$ . Herein,  $\mathbf{a}(\theta_k)(\in \mathbb{C}^M)$  indicates the steering vector of  $k \in \{1, \dots, K\}$ -th satellite. When

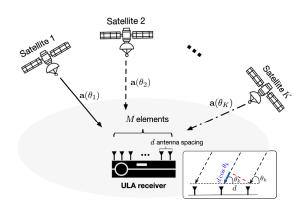


Fig. 1. System model for the receiver with ULA to estimate the DOAs of K satellites.

considering ULA as a receive array antenna, the  $m \in \{1, \dots, M\}$ -th element of the steering vector  $a_m(\theta_k)$  is defined as

$$a_m(\theta_k) = e^{-j\frac{2\pi}{\lambda}d(m-1)\cos\theta_k},\tag{2}$$

where  $\lambda$  means the wavelength and d denotes antenna spacing as illustrated in Fig. 1. Also,  $\mathbf{x} (\in \mathbb{R}^K)$  in (1) means the received signal vector from K satellites. In this paper, to evaluate the effect of the direction finding performance caused only by additive noise, it is assumed that both Doppler frequency and carrier frequency are compensated in advance. In other words,  $x_k$ , the k-th element of  $\mathbf{x}$ , represents the signal power of k-th satellite signal, i.e.,  $x_k = \sqrt{P_k}$  where  $P_k$  denotes the power of k-th satellite signal. Finally,  $\mathbf{w} (\in \mathbb{C}^M)$  denotes the additive white Gaussian noise vector, and it is assumed that  $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_M)$ .

## III. PROPOSED CORRELATIVE INTERFEROMETER TECHNIQUE WITH MULTI-SAMPLE DIVERSITY EFFECT

In this section, we propose a novel CI technique to jointly utilize spatial features in multiple time-domain samples. First of all, for the proposed CI technique with multi-sample diversity, the receiver collects T samples. Here, it is assumed that the direction of the incident signals does not change during the T sample period. Then, the received signal matrix  $\mathbf{Y} (\in \mathbb{C}^{M \times T})$  for T time samples is expressed as

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{W},\tag{3}$$

where, **Y** consists of *T* columns in which the  $t(\in \{1, \dots, T\})$ -th column denotes the received signal vector  $\mathbf{y}_t (\in \mathbb{C}^M)$  at *t*-th time,  $\mathbf{X} (\in \mathbb{R}^{K \times T})$  denotes a signal matrix for *K* satellites and *T* samples, and  $\mathbf{W} (\in \mathbb{C}^{M \times T})$  means the additive noise matrix for *T* samples.

The conventional CI algorithm can simply be performed by measuring the phase of the received signal incident on each antenna element and comparing it with the candidate phases. Specifically, if the signal received by the *m*-th antenna element is  $y_m$ , the phase of  $y_m$  is denoted as  $r_m$  in this paper, i.e.,

## Algorithm 1 CI with multi-sample for DoA estimation

**Require:** Received signal  $\mathbf{Y}$ , The number of satellites K**Ensure:** Estimated DoA  $\Lambda$ 1: Initialization :  $\Lambda = \emptyset$ 2: for  $\hat{\theta} \in [0, \pi]$  do for  $t = 1, \cdots, T$  do 3: for  $m = 1, \cdots, M$  do 4:  $r_{m,t} = \angle y_{m,t}$   $c_m(\hat{\theta}) = -\frac{2\pi}{\lambda}d(m-1)\cos\hat{\theta}$ end for  $J_t(\hat{\theta}) = \sum_{m=1}^M \cos[c_m(\hat{\theta}) - r_{m,t}].$ 5: 6: 7: 8: end for  $J(\hat{\theta}) = \sum_{t=1}^{T} J_t(\hat{\theta})$ 9: 10: 11: end for 12:  $\Lambda \leftarrow$  Angles corresponding to K peaks in  $J(\hat{\theta})$ . 13: **Return**  $\Lambda$ 

 $r_m = \angle y_m$ . And, from (2), the candidate phase  $c_m(\hat{\theta})$  for an arbitrary direction  $\hat{\theta} \in [0, \pi]$  can be defined as

$$c_m(\hat{\theta}) = -\frac{2\pi}{\lambda} d(m-1)\cos\hat{\theta}.$$
 (4)

Then, by checking the difference between the measured received signal and all candidate phases, the direction with the smallest difference should be selected. Our proposed CI technique is intended to efficiently utilize the difference between the phase of the received signal and ideal candidate phases over all time samples to average out the effects of noise. Hence, letting the *m*-th element of  $\mathbf{y}_t$  be  $y_{m,t}$  and the phase of  $y_{m,t}$  be  $r_{m,t}$ , i.e.,  $r_{m,t} = \angle y_{m,t}$ , the temporal cost function  $J_t(\hat{\theta})$  for an arbitrary angle  $\hat{\theta}$  at *t*-th sample can be defined as

$$J_t(\hat{\theta}) = \sum_{m=1}^M \cos[c_m(\hat{\theta}) - r_{m,t}].$$
 (5)

Here, the cost function, which is the criterion for determining the smallest phase difference, can exist in various forms [12]. In this paper, we utilize the cosine function as a cost function since it has the best performance among cost functions in [12]. Finally, by adding the temporal cost functions for all samples, the cost function of the proposed CI can be calculated as

$$J(\hat{\theta}) = \sum_{t=1}^{T} J_t(\hat{\theta}) = \sum_{t=1}^{T} \sum_{m=1}^{M} \cos[c_m(\hat{\theta}) - r_{m,t}].$$
 (6)

After that, DoA estimation is completed by finding K peaks in the cost function. The overall process of the proposed CI technique is summarized as Algorithm 1.

## **IV. SIMULATION RESULTS**

In this section, we validate the effectiveness of the proposed CI technique and compare it with both the conventional CI method and the classical subspace-based MUSIC method. We consider the receive array antenna as a ULA with 15 elements in which antenna spacing is set to half a wavelength, i.e.,

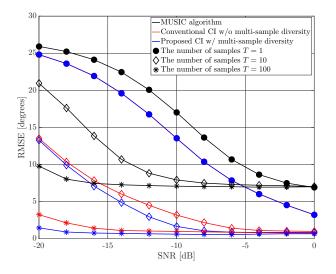


Fig. 2. RMSE performance according to SNR when K = 3 and angular information is  $40^{\circ}$ ,  $60^{\circ}$ , and  $120^{\circ}$ , respectively.

 $d = \lambda/2$ . And, we set the carrier frequency to 12.7GHz. In addition, considering the high path attenuation between the satellite and the ground, the simulation was conducted for the low SNR regime. It is also assumed that the number of satellites is 3 where the angle of signals is set to 40°, 60°, and 120°, respectively. The angular resolution for the direction-finding techniques is 1°. In this paper, as a performance metric, the root mean squared error (RMSE) is considered to evaluate the direction finding performance through Monte-Carlo simulations as follows

$$\text{RMSE} = \sqrt{\frac{1}{NK} \sum_{n=1}^{N} \sum_{k=1}^{K} \left[ \left( \hat{\theta}_{k,n}^{*} - \theta_{k,n} \right)^{2} \right]}, \quad (7)$$

where  $\theta_{k,n}$  and  $\hat{\theta}_{k,n}^*$  denote the true angle and the selected angle from the direction-finding algorithm of the *k*-th satellite in the *n*-th iteration, respectively.

Fig. 2 shows the RMSE performance of the proposed CI technique according to SNR over multiple time-domain samples. For the conventional CI method, the mean value of the received signal over all samples is used for DOA estimation. The RMSE of all DoA estimation techniques tends to decrease as the number of time-domain samples increases. Since the proposed CI technique is regarded as a generalized algorithm of the conventional CI to multiple samples, it can be seen that the proposed CI obtains the same performance as the conventional CI for a single sample. However, as for gathering multiple signals at the same time, when estimating one DoA, the directional components of other signals may interfere and it makes the accurate direction finding to be hard. Nevertheless, it is confirmed that the proposed CI technique yields better RMSE performance than the conventional CI technique and MUSIC method by jointly combining spatial features of signal components for each sample, regardless of the number of time-domain samples and operating SNR values. We called

this performance-enhancing effect the multi-sample diversity effect.

## V. CONCLUSION

In this paper, we proposed a novel multi-sample diversitybased correlative interferometer (CI) technique for precise direction-of-arrival (DoA) estimation, which can be regarded as a generalized form of the conventional CI technique. The proposed CI technique allows the receiver to jointly combine the spatial characteristics from each time-domain sample, improving the direction finding performance. Through computer simulations, we verified that proposed CI elaborately estimates the DoA of satellite signals with high precision. Moreover, we showed that the proposed CI technique significantly outperforms the classical subspace-based MUSIC algorithm in terms of DoA estimation performance.

## ACKNOWLEDGMENT

This work was supported in part by by Institute of Information & communications Technology Planning & Evaluation (IITP) through Development of Incumbent Radio Stations Protection and Frequency Sharing Technology through Spectrum Challenge funded by the Korea government (MSIT) under Grant No. 2019-0-00964 and in part by the Intelligent Technology Development Program on Disaster Response and Emergency Management under Grant No. 2022-MOIS37-005.

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